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Design of Transonic Compressor Cascades Using Hodograph Method

Chen Zuoyi Guo Jingrong
(Dept. of Thermal Engineering, Tsinghua University)
(Beijing, China)

1. Introduction

The design of transonic turbine cascade using Hodograph Mathod is presented in Ref.1, 2, 3, etc. But up to now, there are no published papers about the design of transonic compressor cascade using Hodograph Mathod. It is given in this article.

The design of flow mode in the transonic compressor cascade must be as follows: (1) the flow in nozzle part should be uniform and smooth. (2) the location of sonic line should be resonable, and (3) aerodynamic character of the flow canal in subsonic region should be met. The rate through cascade may be determined by velocity distribution in subsonic region. (i.e. by the numerical solution of Chaplygin equation). The supersonic sections A′C′ and AD are determined by the analystical solution of Mixed−Type Hodograph equation. If the shock wave exist that we should consider the flow turn by the shock wave. (The ″shock wave−Mean Stream Line Turn Method″ has been used).

2. The compressor canal design using the analytical solution of Mixed−Type Hodograph equation

In general, the analytical solution of Mixed−Type Hodograph equation (i.e. the nozzle solution) is used in the design of transonic turbine cascade. Can it be used in the design of compressor? Our research shows that it is sure.

For example, if we use the generalization Tricomi approximation, the approximate compressible function is:

$$K_a(\sigma) = b^6(\sigma) / (1 - cb\sigma)^5 \tag{1}$$

For this approximation, comparing the approximate compressible function $K_a(\sigma)$ to the true compressible function $K(\sigma)$, the difference is very small in supersonic region but not in subsonic region. This is suitable for the design of transonic turbine cascade because the analytical solution only apply in the supersonic region. But for the design of compressor cascade, the analytical solution must be used in subsonic region. In order to decrease the difference, we should used the other boundary condition to determine the coefficients b and c, this is:

$$K_a(\sigma)|_{M\sigma^* = 0.3} = K(\sigma)|_{M\sigma^* = 0.3} \tag{2}$$

$$\left.\frac{dK_a(\sigma)}{d\sigma}\right|_{\sigma=0} = \left.\frac{dK(\sigma)}{d\sigma}\right|_{\sigma=0} \tag{3}$$

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and we obtain the c, b as follows:

If $\gamma = 1.4$ $\quad c = -0.28236$

$b = 1.15709$

The comparision between $K(\sigma)$ and $Ka(\sigma)$ is shown in Fig.2.

3. To determine the leading edge shock wave and coming flow parameters of transonic compressor cascade

On the basis of the sonicline location which has been given in design and according to the application of analytical solution in compressor canal, we may obtain the profile C′A′ and DA, along with the velocity distribution in C′A′ and DA. (i.e. obtain the geometry of leading edge and the Ma_A, $Ma_{A'}$(see Fig.3)). If the computation model in Fig.3 is used, we can consider that the flow turn proceeded from coming flow Ma_{100} at the leading edge of suction and pressure surface respectively. If the turn angles are δ and δ',

thus $\quad \delta + \delta' = \Phi$

here Φ is geometric angle of leading edge.

Two shock waves are produced when the flow turn suddenly. The shock wave angles are β_1, β_2, β'_1 and β'_2 respectively. Thus, from the relationship of oblique shock wave, we obtain:

$$tg(\beta_1 - \beta_2) = 2ctg\beta_1\left[Ma_1^2 sin^2\beta_1 - 1\right] / \left[Ma_1^2(\gamma + cos2\beta_1) + 2\right] \tag{4}$$

$$tg(\beta'_1 - \beta'_2) = 2ctg\beta'_1\left[Ma_1^2 sin^2\beta'_1 - 1\right] / \left[Ma_1^2(\gamma + cos2\beta'_1) + 2\right] \tag{5}$$

$$\beta'_1 - \beta'_2 = \Phi - (\beta_1 - \beta_2) \tag{6}$$

$$Ma_2^2 = \frac{Ma_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} Ma_1^2 sin^2\beta_1 - 1} + \frac{Ma_1^2 cos^2\beta_1}{\frac{\gamma - 1}{2} Ma_1^2 sin^2\beta_1 + 1} \tag{7}$$

$$Ma_2^2 = \frac{Ma_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} Ma_1^2 sin^2\beta'_1 - 1} + \frac{Ma_1^2 cos^2\beta'_1}{\frac{\gamma - 1}{2} Ma_1^2 sin^2\beta'_1 + 1} \tag{8}$$

in previous five equations, the known parameters are Ma_2, Ma_2' and Φ, and unknown parameters are β_1, β_2, β'_1, β'_2, Ma_1, so the solutions are completely dertermined by the five equations.

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If the shock wave is strong, the flow turn passing the shock wave should be considered. The model of design is shown in Fig.4.

We consider that the shock wave strength is determined by the flow turn on the meanstreamline, i.e. the shock wave angles β_1^* and β_2^* are determined by the flow turn angle δ^*, thus the follwoing relations may be obtained:

$$M_{a_{B1}}^2 = \frac{2ctg\beta_1^* + 2tg(\beta_1^* - \beta_2^*)}{2ctg\beta_1^* sin^2\beta_1^* - tg(\beta_1^* - \beta_2^*)(\gamma + cos2\beta_1^*)} \tag{9}$$

$$M_{a_{B2}}^2 = \frac{Ma_{B1}^2 + \dfrac{2}{\gamma - 1}}{\dfrac{2\gamma}{\gamma - 1} Ma_{B1}^2 sin^2\beta_1^* - 1} + \frac{Ma_{B1}^2 cos^2\beta_1}{\dfrac{\gamma - 1}{2} Ma_{B1}^2 sin^2\beta_1^* + 1} \tag{10}$$

Now, the flow field can be divided into two parts, the region before the shock wave and after the shock wave. In the region A′BA, applied the Hodograph Mixed−Type Equation to determine the profile and velocity distribution of the C′A′ and DB. There are four parameters Ma_{B1}, Ma_{B2}, β_1^*, and β_2^* in the Eq.9 and Eq.10. If the Ma_{B2} is determined, thus the relation between β_1^* and β_2^* can also be determined, and the design of Hodograph Method that consider the sudden change proceeded from the shock wave may be solved with the alternative method. For example, we assume the β_2^* and therefore the location of shock wave can be determined, i.e. the location of point B is determined, and the Ma_{B2} can also be determined. From the Eq.9 and Eq.10, the Ma_{B1} and β_1^* may be obtained, and the δ^* may also be obtained. After this, we can obtain the turn−meanline EF. On the basis of the EF we can solve the velocity distribution on the BA, and the Ma_2 can be obtained. So that from the Eq.4∼ Eg.8 the new β_2^* can be determined. Put the new β_2^* in previous calculation and till it is satisfied.

In the subsonic region, the numerical solution of Chaplygin Eguation can be used. i.e.

$$\frac{\partial}{\partial\theta}\left(P\frac{\partial\psi}{\partial\theta}\right) + \frac{\partial}{\partial M_a^*}\left(Q\frac{\partial\psi}{\partial M_a^*}\right) = -\frac{Q}{M_a^*} L(\psi_\infty) \tag{11}$$

4. Design example

Using previous theory and mathod, two compressor cascade have been designed. The one is called J−3−Type cascade, and the other is called Ma−Type cascade which is designed on the basis of velocity distribution of the compressor cascade which is provided by Ref.4.

The designed parameters of transonic compressor−3 profil are: Mach number at outlet is 0.62, flow angle at outlet is 45 °, cascade pitch is 50mm, the Mach number and flow angle

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at inlet are 50 ° and 1.22 respectively. The designed cascade profile is shown in Fig 5. The profile coordinates and its velocity distribution are shown in Table 1.

Comparing the velocity distribution of J–3–Type cascade with the calculating of the Time Marching method, we find the results are in agreement (see Fig.6).

The profile of Ma–Type cascade comparing with the cascade in Ref.4 is also in agreement (see Fig.7).

5. Conclusion

1. The Hodograph Method may be used to the design of transonic compressor cascade.
2. The flow field may be divided into two parts for using Hodograph method design, to solve the Hodograph Mixed–Teyp Equation in supersonic region and the Chaplygin Equation in subsonic region.
3. If the strength of shock wave is large, the flow turn by shock wave, the "Shock wave–Meanstreamline turn Method" is suitable for this design.

Reference

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3. G.Karadimas, Institute for Fluid Dynamics lecture Series 59.
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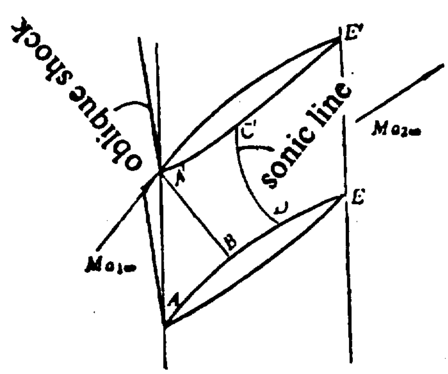

Fig.1 The flow model of
transonic compressor cascade

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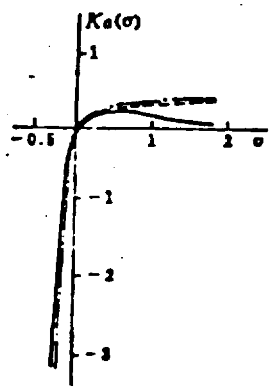

Fig.2 Comparision between Ka(σ) and K(σ) respecting to various coefficients a and b

true K(σ)
Ka(σ): b=0.024 c=−0.78 / b
Ka(σ): b=1.15609 c=−0.28236

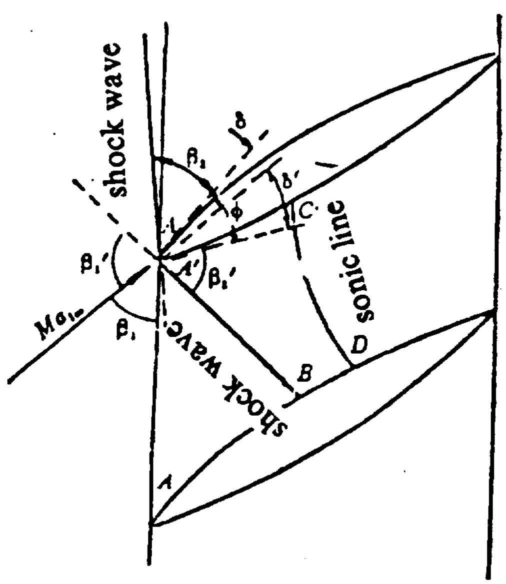

Fig.3 The relation between the leading edge shock wave and coming flow parameters

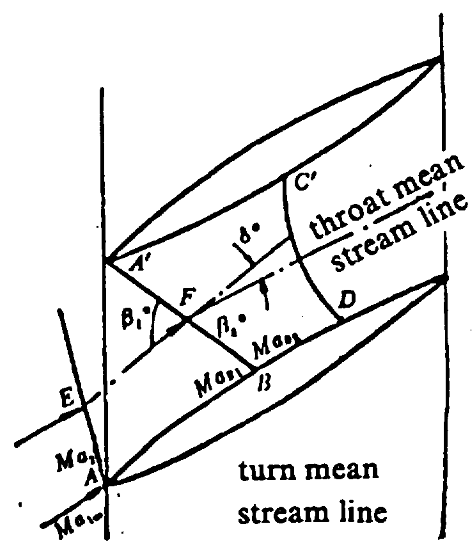

Fig.4 Calculation model in compressor cascade

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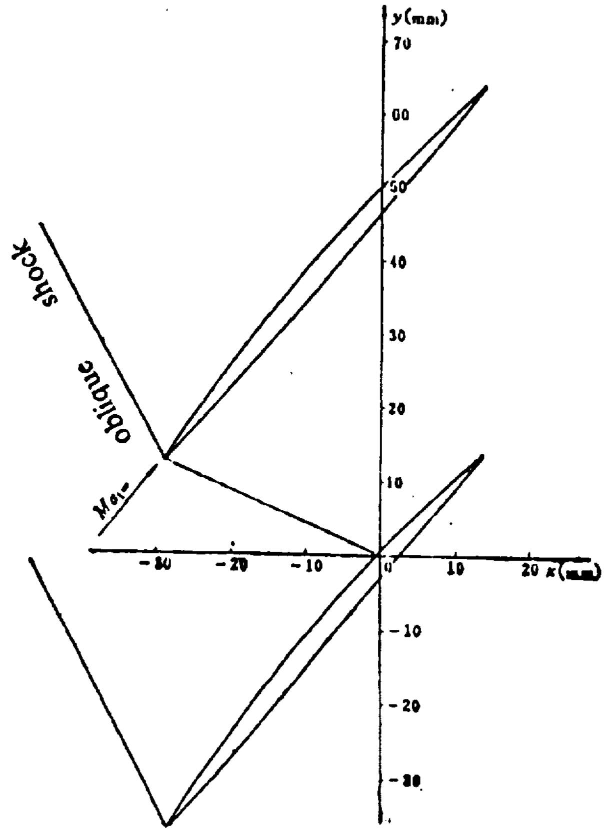

Fig.5 Transonic compressor cascade J–3–Type with Hodograph Method oblique shock

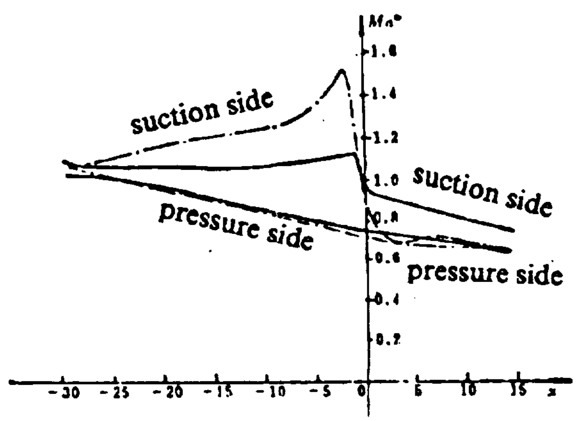

Fig.6 Velocity distribution comparison between Hodograph Method and Time Marching method

——— Hodograph Method
—·— Time Marching Method

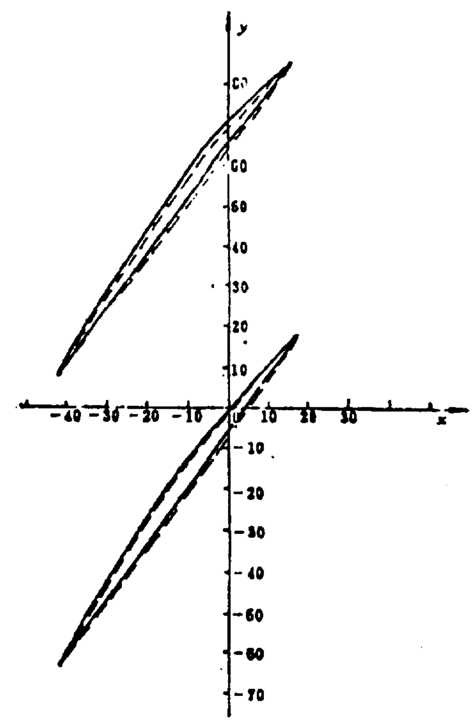

Fig.7 Comparison between the transonic compressor cascade (Ma–Type) proceeded from inverted calculation with Hodograph Method and Ref.4

——— Profile in Ref.4
- - - - Profile by Hodograph Method